

Low-Complexity RIS Phase Error Estimation Method for RIS-Aided OFDM Systems

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ABSTRACT

Reconfigurable intelligent surface (RIS) has been recently considered a key technology for future wireless communication systems. In order to achieve the optimal performance that RIS can provide, accurate channel information estimation is essential. However, existing RIS-based techniques did not consider RIS phase shift error. RIS inevitably has phase shift error due to hardware imperfection/defects or quantized phase shifts. In this paper, we propose a phase shift error estimation method with low computational complexity and high accuracy. We demonstrate efficiency of the proposed RIS phase shift error estimation method through simulation results and comparison with a benchmark method.

Key Words : Reconfigurable intelligent surface, RIS, Phase synchronization, Phase shift error

I. Introduction

The wireless channel is usually considered to be an uncontrollable entity. This limits the performance of wireless communications. To circumvent this problem, there is a new approach for controlling wireless channels based on the use of *reconfigurable intelligent surfaces* (RISs)^[1]. An RIS is a two-dimensional metasurface consisting a large number of passive reflecting elements. Each of RIS elements can control/adjust the reflections of incoming wireless signals to target a desirable performance metric such as the achievable rate^[2]. RIS-assisted wireless communications can significantly reduce the system hardware cost.

In order to achieve the optimal performance of the RIS, the optimization of the RIS reflection coefficients requires precise channel state information (CSI) estimation, which is relatively hard to obtain in the absence of active elements on the RIS. To tackle this problem, several channel estimation methods for

RIS-assisted wireless communications have recently been proposed^[3-6]. However, these existing methods do not consider the RIS phase shift errors.

In the conventional method for RIS-aided networks, it is assumed that there is no phase shift error^[7]. However, because of the hardware imperfection, the phase shift error can degrade the performance. Against this background, we propose a low-complexity RIS phase shift error estimation method. The RIS phase shift errors are estimated using a least-squares based approach. With numerical results, we show that the proposed estimation method has a significantly lower computational complexity while showing a good performance compared to the benchmark method.

Notation: Superscripts $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ denote matrix transpose, Hermitian transpose, and inversion operations, respectively. $\|A\|$, $((a))_b$, and I_p denote the Frobenius norm of A , the operation of a modulo b , and a $p \times p$ identity matrix, respectively.

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II. System Model

In this paper, we consider an RIS-assisted OFDM system with N subcarriers transmitting over frequency-selective fading channels. As shown in Fig. 1, the RIS is deployed to enable wireless communication from one single-antenna user to a single-antenna base station (BS). The RIS consists of R passive reflecting elements, each of which can independently adjust the phase of the reflected signal. Here, we assume that the line-of-sight (LoS) link between the user and the BS is blocked by obstacles.

The uplink (UL) frequency-domain received signal at the BS antenna on subcarrier n for pilot block b can be written as

$$\begin{aligned} y_b^{\text{UL}}(n) &= x_b^{\text{UL}}(n) \sum_{r=0}^R h_r^{\text{UL}}(n) \phi_{r,b}^{\text{UL}} + w_b^{\text{UL}}(n) \\ &= x_b^{\text{UL}}(n) \sum_{r=0}^R h_r^{\text{UL}}(n) e^{j2\pi(\theta_{r,b}^{\text{UL}} + \varepsilon_r)} + w_b^{\text{UL}}(n), \end{aligned} \quad (1)$$

where $\theta_{r,b}^{\text{UL}} \in [-\pi/2, \pi/2)$ is the normalized phase shift on RIS elements r for UL pilot block b , $\varepsilon_r \in [-\pi, \pi)$ is the normalized phase shift offset on RIS elements r , $x_b^{\text{UL}}(n)$ is the OFDM symbol on subcarrier n for UL pilot block b , $h_r^{\text{UL}}(n)$ is the UL channel frequency response (CFR) from the user to the BS antenna via RIS element r on subcarrier n , and $w_b^{\text{UL}}(n)$ is the frequency-domain circularly-symmetric complex additive white Gaussian noise (AWGN) at

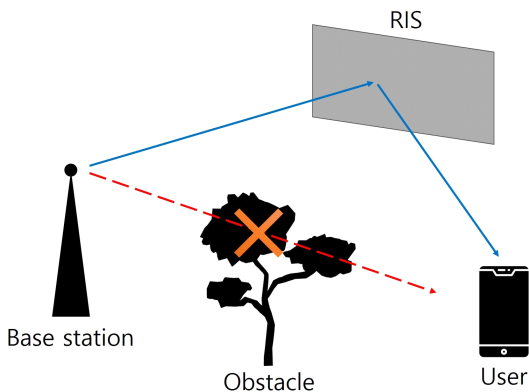


Fig. 1. RIS-aided wireless communication system.

the BS antenna on subcarrier n for UL pilot block b having zero mean and variance σ^2 . Here, we define vectors $s_b^{\text{UL}} = [s_b^{\text{UL}}(0), s_b^{\text{UL}}(1), \dots, s_b^{\text{UL}}(N-1)]^T$, $h_r^{\text{UL}} = [h_r^{\text{UL}}(0), h_r^{\text{UL}}(1), \dots, h_r^{\text{UL}}(N-1)]^T \in \mathbb{C}^{N \times 1}$, and $w_b^{\text{UL}} = [w_b^{\text{UL}}(0), w_b^{\text{UL}}(1), \dots, w_b^{\text{UL}}(N-1)]^T \in \mathbb{C}^{N \times 1}$.

Consequently, the UL received vector at the BS antenna for pilot block b can be written as $y_b^{\text{UL}} = [y_b^{\text{UL}}(0), y_b^{\text{UL}}(1), \dots, y_b^{\text{UL}}(N-1)]^T$.

The corresponding downlink (DL) received signal at the user on subcarrier n for pilot block b can be written as

$$\begin{aligned} y_b^{\text{DL}}(n) &= x_b^{\text{DL}}(n) \sum_{r=0}^R h_r^{\text{DL}}(n) \phi_{r,b}^{\text{DL}} + w_b^{\text{DL}}(n) \\ &= x_b^{\text{DL}}(n) \sum_{r=0}^R h_r^{\text{DL}}(n) e^{j2\pi(\theta_{r,b}^{\text{DL}} + \varepsilon_r)} + w_b^{\text{DL}}(n), \end{aligned} \quad (2)$$

where $\theta_{r,b}^{\text{DL}} \in [-\pi/2, \pi/2)$ is the normalized phase shift on RIS elements r for DL pilot block b , $s_b^{\text{DL}}(n)$ is the OFDM symbol on subcarrier n for DL pilot block b , $h_r^{\text{DL}}(n)$ is the DL CFR from the BS antenna to the user via RIS element r on subcarrier n , and $w_b^{\text{DL}}(n)$ is the frequency-domain circularly-symmetric complex AWGN at the BS antenna on subcarrier n for DL pilot block b having zero mean and variance σ^2 .

When time division duplex (TDD) is used and we assume the channel reciprocity as the propagation channel is essentially reciprocal so that the corresponding DL channel is the Hermitian transpose of the UL channel^[8,9], i.e.,

$$h_r^{\text{DL}}(n) = (h_r^{\text{UL}}(n))^*. \quad (3)$$

Consequently, the DL received signal at the user on subcarrier n for pilot block b can be rewritten as

$$y_b^{\text{DL}}(n) = x_b^{\text{DL}}(n) \sum_{r=0}^R h_r^*(n) e^{j2\pi(\theta_{r,b}^{\text{DL}} + \varepsilon_r)} + w_b^{\text{DL}}(n), \quad (4)$$

where $h_{r,m}^{\text{UL}}(n) = h_{r,m}(n)$.

III. Proposed RIS Phase Shift Error Estimation Method

The BS transmits N -length pilot sequences R times for DL wireless communications. After that, the user transmits N -length pilot sequences R times for UL wireless communications.

The UL received vector at the BS antenna on sub-carrier n over all b can be represented as

$$\begin{aligned}
 y^{\text{UL}}(n) &= \begin{bmatrix} y_1^{\text{UL}}(n) & y_2^{\text{UL}}(n) & \dots & y_R^{\text{UL}}(n) \end{bmatrix} \\
 &= \begin{bmatrix} h_1(n)e^{j2\pi\epsilon_1} & h_2(n)e^{j2\pi\epsilon_2} & \dots & h_R(n)e^{j2\pi\epsilon_R} \end{bmatrix} \times \\
 &\quad \begin{bmatrix} x_1^{\text{UL}}(n)e^{j2\pi\theta_{1,1}^{\text{UL}}} & x_2^{\text{UL}}(n)e^{j2\pi\theta_{1,2}^{\text{UL}}} & \dots & x_R^{\text{UL}}(n)e^{j2\pi\theta_{1,R}^{\text{UL}}} \\
 x_1^{\text{UL}}(n)e^{j2\pi\theta_{2,1}^{\text{UL}}} & x_2^{\text{UL}}(n)e^{j2\pi\theta_{2,2}^{\text{UL}}} & \dots & x_R^{\text{UL}}(n)e^{j2\pi\theta_{2,R}^{\text{UL}}} \\
 \vdots & \vdots & \ddots & \vdots \\
 x_1^{\text{UL}}(n)e^{j2\pi\theta_{R,1}^{\text{UL}}} & x_2^{\text{UL}}(n)e^{j2\pi\theta_{R,2}^{\text{UL}}} & \dots & x_R^{\text{UL}}(n)e^{j2\pi\theta_{R,R}^{\text{UL}}} \end{bmatrix} \\
 &\quad + \begin{bmatrix} w_1^{\text{UL}}(n) & w_2^{\text{UL}}(n) & \dots & w_R^{\text{UL}}(n) \end{bmatrix} \\
 &= \tilde{h}^{\text{UL}}(n)\Theta_{\text{UL}}(n) + w^{\text{UL}}(n). \tag{5}
 \end{aligned}$$

By setting $\Theta_{\text{UL}}^H(n)\Theta_{\text{UL}}(n) = \frac{1}{R}I_R$, we can obtain

$$\begin{aligned}
 \tilde{y}^{\text{UL}}(n) &= \begin{bmatrix} \tilde{y}_1^{\text{UL}}(n) & \tilde{y}_2^{\text{UL}}(n) & \dots & \tilde{y}_R^{\text{UL}}(n) \end{bmatrix} \\
 &= y^{\text{UL}}(n)\Theta_{\text{UL}}^{-1} \\
 &= \tilde{h}^{\text{UL}}(n) + w^{\text{UL}}(n)\Theta_{\text{UL}}^{-1} \\
 &= \tilde{h}^{\text{UL}}(n) + \tilde{w}^{\text{UL}}(n). \tag{6}
 \end{aligned}$$

By designing $x_b^{\text{DL}}(n) = x(n)$, (4) can be written as

$$y_b^{\text{DL}}(n) = x(n) \sum_{r=0}^R h_r^*(n) e^{j2\pi(\theta_{r,b}^{\text{DL}} + \epsilon_r)} + w_b^{\text{DL}}(n), \tag{7}$$

The DL received vector on subcarrier n over all b can be represented as

$$\begin{aligned}
 y^{\text{DL}}(n) &= \begin{bmatrix} y_1^{\text{DL}}(n) & y_2^{\text{DL}}(n) & \dots & y_R^{\text{DL}}(n) \end{bmatrix} \\
 &= \begin{bmatrix} h_1^*(n)e^{j2\pi\epsilon_1} & h_2^*(n)e^{j2\pi\epsilon_2} & \dots & h_R^*(n)e^{j2\pi\epsilon_R} \end{bmatrix} \times \\
 &\quad \begin{bmatrix} x(n)e^{j2\pi\theta_{1,1}^{\text{DL}}} & x(n)e^{j2\pi\theta_{1,2}^{\text{DL}}} & \dots & x(n)e^{j2\pi\theta_{1,R}^{\text{DL}}} \\
 x(n)e^{j2\pi\theta_{2,1}^{\text{DL}}} & x(n)e^{j2\pi\theta_{2,2}^{\text{DL}}} & \dots & x(n)e^{j2\pi\theta_{2,R}^{\text{DL}}} \\
 \vdots & \vdots & \ddots & \vdots \\
 x(n)e^{j2\pi\theta_{R,1}^{\text{DL}}} & x(n)e^{j2\pi\theta_{R,2}^{\text{DL}}} & \dots & x(n)e^{j2\pi\theta_{R,R}^{\text{DL}}} \end{bmatrix} \\
 &\quad + \begin{bmatrix} w_1^{\text{DL}}(n) & w_2^{\text{DL}}(n) & \dots & w_R^{\text{DL}}(n) \end{bmatrix} \\
 &= \tilde{h}^{\text{DL}}(n)\Theta_{\text{DL}}(n) + w^{\text{DL}}(n). \tag{8}
 \end{aligned}$$

By setting $\Theta_{\text{DL}}^H(n)\Theta_{\text{DL}}(n) = \frac{1}{R}I_R$, we can obtain

$$\begin{aligned}
 \tilde{y}^{\text{DL}}(n) &= \begin{bmatrix} \tilde{y}_1^{\text{DL}}(n) & \tilde{y}_2^{\text{DL}}(n) & \dots & \tilde{y}_R^{\text{DL}}(n) \end{bmatrix} \\
 &= y^{\text{DL}}(n)\Theta_{\text{DL}}^{-1} \\
 &= \tilde{h}^{\text{DL}}(n) + w^{\text{DL}}(n)\Theta_{\text{DL}}^{-1} \\
 &= \tilde{h}^{\text{DL}}(n) + \tilde{w}^{\text{DL}}(n). \tag{9}
 \end{aligned}$$

By multiplying $\tilde{y}_r^{\text{UL}}(n)$ and $\tilde{y}_r^{\text{DL}}(n)$, we can obtain

$$\begin{aligned}
 \tilde{y}_r^{\text{UL}}(n)\tilde{y}_r^{\text{DL}}(n) &= |h_r(n)|^2 e^{j4\pi\epsilon_r} \\
 &\quad + \tilde{w}_r^{\text{UL}}(n)h_r^*(n)e^{j2\pi\epsilon_r} \\
 &\quad + \tilde{w}_r^{\text{DL}}(n)h_r(n)e^{j2\pi\epsilon_r} \\
 &\quad + \tilde{w}_r^{\text{UL}}(n)\tilde{w}_r^{\text{DL}}(n). \tag{10}
 \end{aligned}$$

Since the expected value of $\tilde{w}_r^{\text{UL}}(n)$ and $\tilde{w}_r^{\text{DL}}(n)$ is 0 for all n , we can obtain the averaged correlation of $\tilde{y}_r^{\text{UL}}(n)$ and $\tilde{y}_r^{\text{DL}}(n)$ over n as following:

$$C_r = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{y}_r^{\text{UL}}(n)\tilde{y}_r^{\text{DL}}(n) \approx \frac{1}{N} \sum_{n=0}^{N-1} |\tilde{h}_r(n)|^2 e^{j4\pi\epsilon_r} \tag{11}$$

Consequently, we can estimate ϵ_r as following

$$\hat{\epsilon}_{4\pi,r} = \frac{\angle(C_r)}{4\pi}. \tag{12}$$

IV. Benchmark Method: ML estimation

In this section, in order to investigate the efficiency of the proposed RIS phase estimation method, the

maximum-likelihood (ML) method is briefly provided. In this paper, the ML method is considered since the same pilot structure can be used for both the proposed method and the ML method, the performance of the ML method with the same pilot structure can provide the optimal performance for the same pilot setup. The other estimation method such as estimation with an RIS on-off method, in which only a single RIS element is active while other RIS elements are off for an assigned time slot, zero-forcing method, etc., require additional pilot resources, i.e., they require different pilot structure, they are not considered in this paper.

When it is assumed that the CFR is known at the BS, we can estimate RIS phase shift error as:

$$\varepsilon = \arg \min_{\varepsilon} \langle \|y_b^{\text{UL}} - \hat{y}_b^{\text{UL}}\|, \|y_b^{\text{DL}} - \hat{y}_b^{\text{DL}}\| \rangle \quad (13)$$

where

$$\hat{\varepsilon} = [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \dots, \hat{\varepsilon}_R],$$

$$\hat{y}_b^{\text{UL}}(n) = x_b^{\text{UL}}(n) \sum_{r=0}^R h_r^{\text{UL}}(n) e^{j2\pi(\theta_{r,b}^{\text{UL}} + \varepsilon_r)},$$

and

$$\hat{y}_b^{\text{DL}}(n) = x_b^{\text{DL}}(n) \sum_{r=0}^R h_r^{\text{DL}}(n) e^{j2\pi(\theta_{r,b}^{\text{DL}} + \varepsilon_r)}.$$

It is worth noticing that the proposed method does not require knowledge of channels.

V. Computational Complexity Comparison

In this section, we analyze and compare the computational complexity of the proposed RIS phase shift error estimation method with the ML estimation method in Section IV in terms of the number of complex multiplications.

The computational complexity of the proposed RIS phase shift error estimation is given by

$$C_{\text{Pro}} = O(R^3 + NR^2), \quad (14)$$

where the computational complexity of inverse matrix operation for Θ_{DL} is given as $O(R^3)$, that of (9) is given as $O(NR^2)$, and that of (10)-(12) is given as $O(NR)$.

The computational complexity of the RIS phase shift error ML estimation method in Section IV is given by

$$C_{\text{ML}} = O((NR^2)N_{\text{iter}}^R), \quad (15)$$

where N_{iter} is the number of candidate for each ε_r , and the computational complexity to obtain $\hat{y}_b^{\text{UL}} = [\hat{y}_b^{\text{UL}}(0), \hat{y}_b^{\text{UL}}(1), \dots, \hat{y}_b^{\text{UL}}(N-1)]^T$, that to obtain $\hat{y}_b^{\text{DL}} = [\hat{y}_b^{\text{DL}}(0), \hat{y}_b^{\text{DL}}(1), \dots, \hat{y}_b^{\text{DL}}(N-1)]^T$, that of the norm operation of a vector of length N and are given as $O(NR^3)$, $O(NR^2)$, and $O(NR)$, respectively. In order to guarantee the performance of the ML estimation, N_{iter} needs to be a large number. Also, in general, for RIS-aided wireless communication systems, R has a large value. Consequently, the proposed RIS phase shift error estimation method shows the better computational complexity efficiency than the ML estimation method in Section IV.

VI. Numerical Results

In this section, we provide simulation results to demonstrate the effectiveness of the proposed RIS phase shift error estimation method. We consider a system with $N = 256$ and $R = 100$. The RIS phase shift error is generated from a uniform distribution within the range $(\pi/2, \pi/2]$. The results are obtained after 5000 independent realizations of the channel and RIS phase shift error.

The MSE performance of the proposed RIS phase shift error estimation method and that of the ML estimation method in Section IV as a function of signal-to-ratio (SNR) is given in Fig. 2. When SNR increases, the performance of the proposed method is closer to the that of the benchmark method.

The MSE performance of the proposed RIS phase shift error estimation method as a function of the length of pilot sequence, N_p is given in Fig. 3. When N increases, the performance of the proposed method

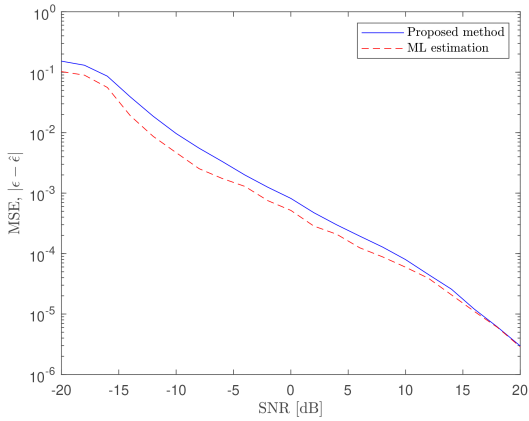


Fig. 2. The performance comparison between the proposed estimation method and the benchmark method as a function of SNR.

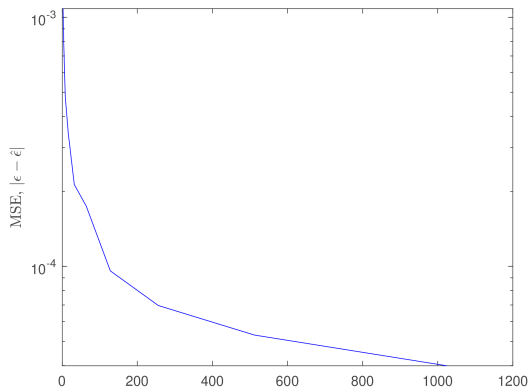


Fig. 3. The performance of the proposed RIS phase error estimation method as a function of N .

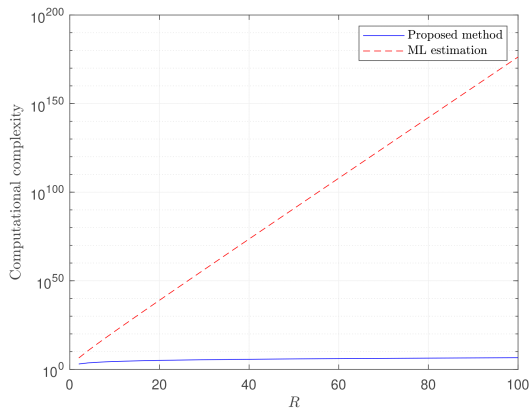


Fig. 4. Computational complexity comparison between the proposed estimation method and the benchmark method as a function of R .

is improved. This is because with a larger value of N , the noise can be more mitigated in (11).

Fig. 4 shows the computational complexity comparison between the proposed RIS phase shift error estimation method and the benchmark ML estimation method. The simulation setup for this figure is $N_{iter} = 50$ and $N = 256$. Compared to the benchmark, the proposed method shows significantly lower computational burden while it shows the comparable estimation accuracy.

VII. Conclusions and Future Works

In this paper, we proposed the low-complexity RIS phase shift error estimation methods based on channel reciprocity. The proposed method considers the different phase shift error on each RIS reflection element. The proposed method exhibits comparable RIS phase shift estimation performance as well as a significantly lower complexity when compared to a benchmark ML estimation method. For the future works, we will expand this method from the scenario with a single-antenna BS and a single-antenna user to the scenario with a multi-antenna BS and single-antenna users.

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